**2017年-2018学年度第一学期**

**华中科技大学本科生课程考试补考试卷**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **课程名称： 运筹学（一）** | **课程类别** | **□公共课**  **■专业课** | **考试形式** | **□开卷**  **■闭卷** |
| **所在院系： 自动化学院 专业及班级： 考试日期：** | | | | |
| **学 号： 姓名： 任课教师：** | | | | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **题号** | **一** | **二** | **三** | **四** | **五** | **总分** |
| **分数** |  |  |  |  |  |  |

|  |  |
| --- | --- |
| **得分** | **评卷人** |
|  |  |

**一、（20分）试求解如下线性规划问题**

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**解：将模型化为如下：**

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**列出初始单纯形表**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 2 | 1 | 3 | 0 | 0 | 0 | -M |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 0 |  | 7 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 7 |
| 0 |  | 8 | 2 | -3 | 5 | 0 | 1 | 0 | 0 | 4 |
| -M |  | 1 | [1] | 0 | -2 | 0 | 0 | -1 | 1 | 1 |
|  |  |  | 2+M | 1 | 3-2M | 0 | 0 | -M | 0 |  |

选择为换入变量，为换出变量，进行迭代得到：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 2 | 1 | 3 | 0 | 0 | 0 | -M |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 0 |  | 6 | 0 | 1 | 3 | 1 | 0 | 1 | -1 | 2 |
| 0 |  | 6 | 0 | -3 | [9] | 0 | 1 | 2 | -2 | 6/9 |
| 2 |  | 1 | 1 | 0 | -2 | 0 | 0 | -1 | 1 |  |
|  |  |  | 0 | 1 | 7 | 0 | 0 | 2 | -M-2 |  |

选择为换入变量，为换出变量，进行迭代得到：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 2 | 1 | 3 | 0 | 0 | 0 | -M |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 0 |  | 4 | 0 | [2] | 0 | 1 | -1/3 | 1/3 | -1/3 |  |
| 3 |  | 2/3 | 0 | -1/3 | 1 | 0 | 1/9 | 2/9 | -2/9 |  |
| 2 |  | 7/3 | 1 | -2/3 | 0 | 0 | 2/9 | -5/9 | 5/9 |  |
|  |  |  | 0 | 10/3 | 0 | 0 | -7/9 | 4/9 | -M-4/9 |  |

选择为换入变量，为换出变量，进行迭代得到：

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 2 | 1 | 3 | 0 | 0 | 0 | -M |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 2 | 0 | 1 | 0 | 1/2 | -1/6 | 1/6 | -1/6 |  |
| 3 |  | 4/3 | 0 | 0 | 1 | 1/6 | 1/18 | 5/18 | -5/18 |  |
| 2 |  | 11/3 | 1 | 0 | 0 | 1/3 | 1/9 | -4/9 | 4/9 |  |
|  |  |  | 0 | 0 | 0 | -5/3 | -2/9 | -5/9 | -M+5/9 |  |

**所有检验数都为复数，得到最优解为：**

**最优值为：**

|  |  |
| --- | --- |
| **得分** | **评卷人** |
|  |  |

**二、（15分）已知线性规划的最优解为*x*\*=（0，0，4，4）T。试利用互补松弛定理求对偶问题最优解。**

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**解：对偶问题为：**

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**由于*x*3\*=*x*4\*=4>0，是松约束，故（2c）与（2d）是紧约束，即对*Y*\*成立等式：**



**把*x*\*代入原问题三个约束中，可知(1c)是松的，故*y*3\*=0，然后解方程组：**

**得到：**



**故对偶最优解为：Y\*=（6/5，1/5，0）, z\*=*w*\*=28**

|  |  |
| --- | --- |
| **得分** | **评卷人** |
|  |  |

**三、（15分）某厂生产三种产品受到两种原材料的限制。为求最大利润，求得最终单纯形表如下表所示。其中*x*4*，x*5为松驰变量。**

**（1）利用最终单纯形表求各产品的单位销售价格*c*1*，c*2*，c*3。**

**（2）*c*3增加到多少，仍能使现行计划保持最优。**

**（3）计算这两种原料的影子价格，如果能以每单位2元的价格在市场上购入更多的原料*b*2，是否合算？又若*b*2的价格为5元呢？**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  |  |  | **0** | **0** |
|  |  |  |  |  |  |  |  |
|  |  | **1** | **1** | **0** | **1** | **3** | **-1** |
|  |  | **2** | **0** | **1** | **1** | **-1** | **2** |
| **-z** | | **8** | **0** | **0** | **-4** | **-3** | **-4** |

解（1）利用最终单纯形表*x*4*, x*5的检验数，

0－3 *c*1＋ *c*2＝－3及 0＋*c*1－2 *c*2＝－4解得，*c*1＝2，*c*2＝3。

利用最终单纯形表*x*3的检验数σ3＝*c*3－*c*1－*c*2＝－4，*c*3＝1。

（2）*c*3为非基变量的目标函数系数，则*c*3的改变只是影响*x*3的检验数，

*σ*3＝*c*3－*c*1－*c*2 = *c*3 －5≤0，*c*3≤5仍能使现行计划保持最优。

（3）两种原料影子价格分别为3和4。若*b*2的市场价格为2，合算；为5，则不合算。

|  |  |
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| **得分** | **评卷人** |
|  |  |

**四．（20分）**已知某运输问题的产销平衡表和单位运价表如表2所示，试求最优的运输调拨方案。

表2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 | 10 | 2 | 3 | 15 | 9 | 25 |
| A2 | 5 | 10 | 15 | 2 | 4 | 30 |
| A3 | 15 | 5 | 14 | 7 | 15 | 22 |
| A4 | 20 | 15 | 13 | M | 8 | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

解：

vogel法确定初始解

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 | 1 |
| A2 | 5 | 10 | 15 | 2 | 4 | 2 |
| A3 | 15 | 5 | 14 | 7 | 15 | 2 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 | 5 | 3 | 10 | 5 | 4 |  |

第一步

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 |  |  |  |  |  | 30 |
| A3 |  |  |  |  |  | 22 |
| A4 |  |  |  |  |  | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

调整行差、列差

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 |  |
| A2 | 5 | 10 | 15 | 2 | 4 | 2 |
| A3 | 15 | 5 | 14 | 7 | 15 | 2 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 | 10 | 5 | 1 | 5 | 4 |  |

第二步：

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 | 20 |  |  |  |  | 30 |
| A3 |  |  |  |  |  | 22 |
| A4 |  |  |  |  |  | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

调整行差、列差

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 |  |
| A2 | 5 | 10 | 15 | 2 | 4 | 2 |
| A3 | 15 | 5 | 14 | 7 | 15 | 2 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 |  | 5 | 1 | 5 | 4 |  |

第三步：

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 | 20 |  |  | 10 |  | 30 |
| A3 |  |  |  |  |  | 22 |
| A4 |  |  |  |  |  | 30 |
| 销量 | 20 | 20 | 30 | 12 | 25 |  |

调整行差、列差

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 | 1 |
| A2 | 5 | 10 | 15 | 2 | 4 |  |
| A3 | 15 | 5 | 14 | 7 | 15 | 2 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 | 10 | 10 | 1 | M | 7 |  |

第三步：

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 | 20 |  |  | 10 |  | 30 |
| A3 |  |  |  | 2 |  | 22 |
| A4 |  |  |  |  |  | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

调整行差、列差

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 | 1 |
| A2 | 5 | 10 | 15 | 2 | 4 | 2 |
| A3 | 15 | 5 | 14 | 7 | 15 | 9 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 | 10 | 10 | 1 |  | 7 |  |

第四步：

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 | 20 |  |  | 10 |  | 30 |
| A3 |  | 18 |  | 2 |  | 22 |
| A4 |  |  |  |  |  | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

调整行差、列差

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 行差 |
| A1 | 10 | 2 | 3 | 15 | 9 | 1 |
| A2 | 5 | 10 | 15 | 2 | 4 | 2 |
| A3 | 15 | 5 | 14 | 7 | 15 | 1 |
| A4 | 20 | 15 | 13 | M | 8 | 5 |
| 列差 |  |  | 1 |  | 7 |  |

第五步，即为初始解：

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 产量 |
| A1 |  |  | 25 |  |  | 25 |
| A2 | 20 |  |  | 10 |  | 30 |
| A3 |  | 18 | 2 | 2 |  | 22 |
| A4 |  |  | 3 |  | 25 | 28 |
| 销量 | 20 | 18 | 30 | 12 | 25 |  |

判断解是不是最优解，用位势法。

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 销地  产地 | B1 | B2 | B3 | B4 | B5 | 位势 |
| A1 | 10，11 | 2，  8 | 3 | 15，  19 | 9，  11 | 0 |
| A2 | 5 | 10，  10 | 15，  6 | 2 | 4，  0 | 6 |
| A3 | 15，  5 | 5 | 14 | 7 | 15，  6 | 11 |
| A4 | 20，  11 | 15，  11 | 13 | M，  M | 8 | 10 |
| 位势 | -1 | -6 | 3 | -4 | -2 |  |

该解已是最优解。

最优值为：z=3\*25+5\*20+2\*10+5\*18+14\*2+7\*2+13\*3+8\*25=566

|  |  |
| --- | --- |
| **得分** | **评卷人** |
|  |  |

**五、（15分）**有甲乙丙丁4个工人，要分别指派他们完成ABCD 不同的4项工作，每人做各项工作所消耗的时间如表4所示。应如何指派工作，才能使总的消耗时间最少？

表4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **工作**  **工人** | **A** | **B** | **C** | **D** |
| **甲** | **4** | **10** | **6** | **7** |
| **乙** | **2** | **7** | **6** | **3** |
| **丙** | **3** | **3** | **4** | **4** |
| **丁** | **4** | **6** | **6** | **3** |

解：

设0-1型决策变量为，其中，=1表示指派第i个工人完成第j项工作，=0表示不指派第i个工人完成第j项工作，i,j=1,2,3,4。第1，2，3，4个工人分别代表甲乙丙丁。第1,2,3,4项工作分别代表ABCD四项工作。记表示第i个工人完成第j项工作所消耗的时间,i,j=1,2,3,4。则指派问题的数学模型为：

采用匈牙利法求解，步骤入下所示。

1. 将矩阵

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 10 | 6 | 7 |
| 2 | 7 | 6 | 3 |
| 3 | 3 | 4 | 4 |
| 4 | 6 | 6 | 3 |

的每行元素都减去该行的最小值，得到

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 6 | 2 | 3 |
| 0 | 5 | 4 | 1 |
| 0 | 0 | 1 | 1 |
| 1 | 3 | 3 | 0 |

1. 将（1）中的结果矩阵的每列都减去该列的最小值，得到

|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 6 | 1 | 3 |
| 0 | 5 | 3 | 1 |
| 0 | 0 | 0 | 1 |
| 1 | 3 | 2 | 0 |

1. 在（2）中的结果矩阵的各行各列中寻找独立0元，并记以⓪。⓪所在行和列的其他0元素记为。得到

|  |  |  |  |
| --- | --- | --- | --- |
| ⓪ | 6 | 1 | 3 |
|  | 5 | 3 | 1 |
|  | ⓪ |  | 1 |
| 1 | 3 | 2 | ⓪ |

1. 独立0元的个数为3<4，还未找到最优解，需要增加0元。将（3）中的结果矩阵中无⓪的行，标记。得到

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ⓪ | 6 | 1 | 3 |  |
|  | 5 | 3 | 1 |  |
|  | ⓪ |  | 1 |  |
| 1 | 3 | 2 | ⓪ |  |

1. 在（4）中的结果矩阵中标记的行中0元所在的列，标记为。得到

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ⓪ | 6 | 1 | 3 |  |
|  | 5 | 3 | 1 |  |
|  | ⓪ |  | 1 |  |
| 1 | 3 | 2 | ⓪ |  |
|  |  |  |  |  |

1. 在（5）的结果矩阵中，标记的列中⓪元所在的行，标记为。得到

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ⓪ | 6 | 1 | 3 |  |
|  | 5 | 3 | 1 |  |
|  | ⓪ |  | 1 |  |
| 1 | 3 | 2 | ⓪ |  |
|  |  |  |  |  |

1. 标记为的行中所有0元所在列都已被标记为。在（6）中的结果矩阵中，将无的行，以及标记为的列划线，得到

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ⓪ | 6 | 1 | 3 |  |
|  | 5 | 3 | 1 |  |
|  | ⓪ |  | 1 |  |
| 1 | 3 | 2 | ⓪ |  |
|  |  |  |  |  |

1. 选取（7）中的结果矩阵中未被划线覆盖的元素中的最小元素，也就是1。将标记的行的所有元素都减去最小元素，再将标记为的列的所有元素都加上最小元素。得到

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ⓪ | 5 | 0 | 2 |  |
|  | 4 | 2 | 0 |  |
|  | ⓪ |  | 1 |  |
| 2 | 3 | 2 | ⓪ |  |
|  |  |  |  |  |

1. 重复（3）的处理。在（8）的结果矩阵中重新寻找独立0元。得到

|  |  |  |  |
| --- | --- | --- | --- |
|  | 5 | ⓪ | 2 |
| ⓪ | 4 | 2 |  |
|  | ⓪ |  | 1 |
| 2 | 3 | 2 | ⓪ |

1. 独立0元的个数为4个，因此，找到最优解。

最优解为：其余都为0。最优值Z==14.

因此，应指派甲完成工作C，乙完成工作A，丙完成工作B，丁完成工作D。此时总耗时最少，为Z=14。